Chapter 14: Inference For Regression (Last Topic !!)

14.1 Inference about the model. - Day 1

OBJ: You will find a conf. int. for the true slope (β) of a LSRL.

Look @ Bottom of pg 780 to Bottom of pg. 782.

$\hat{y} = a + bx$ (*) Slope b and y-int are statistics - they are calculated from the sample Data. (Like $\bar{x}$ + $\hat{p}$)

(*) To Do Inference we think of a and b as estimators of parameters α and β (Like μ + p)

Conditions for Regression Inference

- Linearity (Check the scatterplot for signs of curvature)
- Independence (check the residual plot for randomness)
- Consistency of Variance (Check Residual Plot to Be sure it is not Fan shaped)
- Normality of Errors (Check the histogram of the residuals)

Inference

- The first step is to estimate the unknown parameters α, β, σ.
- Calculate $\hat{y} = a + bx$
- b of the LSRL is an unbiased estimator of the true slope β.
- a of the LSRL is an unbiased estimator of the true y-int α.

Example 14.2 + 2 ¶ After $\}$ Residuals = OBS y - Exp. y $= y - \hat{y}$

Standard Error About the LSRL

$$s = \sqrt{\frac{\Sigma \text{Residual}^2}{n-2}} = \sqrt{\frac{\Sigma (y-\hat{y})^2}{n-2}}$$ $\}$ ON FS. (In num. of S_{b_1} - St. Dev) use s to estimate the unknown σ.

df of s is n-2 (Why n-2 instead of n-1?)
b/c now we are observing 2-variables!

Conf. Int. For $\beta = b \pm t^* \frac{s}{\sqrt{\Sigma(x-\bar{x})^2}}$

-or-

$b \pm t^* SE$ ...

Example 14.4

HW: 14.10, 14.11

14.1 Inference About the Model - Day 2

__OBJ__: You will Test the hypothesis of Various $H_0 : \beta = ?$

$$t = \frac{b - \beta}{SE_b}$$

Evaluate t by hand.
Use table C to find p-value.

$H_0 : \beta = 0$
- The mean of y does not Δ at all when X Δ
- There is no true Lin. Rel. between x + y.
- $\boxed{\text{Exp Var (x)}}$ has no straight line relationship with $\boxed{\text{resp. var (y).}}$

2007 $\begin{cases} H_0 : \beta = 1 \\ H_a : \beta > 1 \end{cases}$ • The mean of y Δ's at the same rate as X

* Regression output (Statistical Software) Give t and Two sided p-value.

Example 14.5

Example 14.6

__HW__: 14.6

Other values For $H_0 : \beta \Rightarrow$

> I have always been under the impression that a hypothesis test on regression slope is for utility of the model,
> in other words that the slope is or is not zero.

I think it is easy to get lulled into that idea, especially b/c the model utility test is so ubiquitous and we rarely (it seems) have an occasion to test a non-zero slope. I think this rarity is probably due to a lack of knowledge of situations that might lead to hypothesis tests (a) other than zero, and (b) other than slope for regression.

Let me see if I can dredge something up...

Problem #1: Test Prep Company

Suppose I have a wiz-bang new method of test prep for the SAT, and I want to guarantee a 25 point increase across the board on the next test. Then I would think a reasonable model would be: NewScore = alpha + beta*OldScore + Error. My null hypotheses would seem to be: alpha = 25 AND beta = 1.

Problem #2: The "cubic" scaling hypothesis

Some biologist or other has told me that for ears of corn of a given hybrid, the volume is proportional to the cube of the length. (This is the standard assumption for organisms of the same species.)

OK, I reason that this means V = kL^(1/3), and I do the appropriate transformation... Now I have:

$$\text{LogV} = \text{alpha} + (1/3)\text{LogL} + \text{Error}.$$

My null hypotheses would seem to be: alpha = 0 AND beta = 1/3.

Problem #3: Assortative mating

It has been commonly observed (at least among arthropod observers) that larger males tend to mate with larger females. Suppose that a theory is posited that males (being males, right ladies??) will pick females (or at least they THINK they are making the decision) smaller than themselves by 10%. Then, for the typical garden variety male Mexican Redknee tarantula (Brachypelma smithi) one would have a model similar to Problem #2 but w/o the transformation. For the male and female leg lengths, we would suppose the following model to be reasonable:

$$\text{FLL} = \text{alpha} + \text{beta} * \text{MLL}$$

My null hypotheses would seem to be: alpha = 0 AND beta = 0.90.